Book of Abstracts
Parametric Optimization and Related Topics XI

Prague, Czech Republic
September 19–22, 2017

paraopt

Parametric Optimization and Related Topics XI
Prague, Czech Republic

Michal Červinka, Václav Kratochvíl
(ÚTIA AV ČR)

September 19-22, 2017

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11TH INTERNATIONAL CONFERENCE ON PARAMETRIC OPTIMIZATION AND RELATED TOPICS

in honor of Jiří Outrata

Organized by:
Institute of Information Theory and Automation,
Czech Academy of Sciences

and

Faculty of Social Sciences,
Charles University

Prague

September 19-22, 2017

FOREWORD

As chairs of the international conference series Parametric Optimization and Related Topics it is our great pleasure to welcome you to the *11th International Conference on Parametric Optimization and Related Topics (ParaoptXI)* in Prague, September 19-22, 2017.

This conference series was founded in 1985 and, since then, usually took place each 2-3 years in different places: the latter seven conferences were held in Enschede (1995), Tokyo (1997), Dubrovnik (1999), Puebla (2002), Cairo (2005), Cienfuegos (2007) and Karlsruhe (2010). We are indebted and thankful to Jürgen Guddat (Humboldt University Berlin) and Hubertus Th. Jongen (RWTH Aachen University) for leading and promoting the Paraopt conference series very successfully as its executive committee for more than twenty years.

We are very glad for the opportunity to dedicate the present conference ParaoptXI to Jiří Outrata on the occasion of his seventieth birthday. The programme for 86 participants from 21 countries is composed of five invited and 77 contributed talks, held in 22 sessions. The program committee is especially thankful to our distinguished invited speakers, René Henrion, Diethard Klatte, Michal Kočvara, Alexander Kruger, and Boris Mordukhovich, as well as to the organizers of the special sessions, Wolfgang Achtziger and Alexandra Schwartz. We also thank the Institute of Information Theory and Automation, Czech Academy of Sciences and the Faculty of Social Sciences, Charles University, for their generous support without which this conference would not be possible.

We are convinced that the local organizing committee around Michal Červinka has prepared an outstanding conference, and we wish you a very pleasant and stimulating ParaoptXI!

Jan-J. Rückmann and Oliver Stein
(paraopt chairs)

JIŘÍ OUTRATA

Doc. Ing. Jiří Outrata, DrSc., a leading expert in nonsmooth variational analysis, celebrates his 70th birthday this year.

He graduated from the Czech Technical University in Prague in 1971 and pursued his PhD studies at the Czechoslovak Academy of Sciences where he received this degree in 1976. Then he obtained a research position at the Institute of Information Theory and Automation of the Czechoslovak Academy of Sciences where he has been working, with several breaks, until now. In particular, during 1991-1994, he worked at the Mathematical Institute of the Bayreuth University and in 1998 and 2001 at the Friedrich Alexander University of Erlangen-Nuremberg participating both times in the research team of Prof. Dr. Jochem Zowe.

Over the years, his scientific activity moved from the numerical solution of optimal control problems over duality theory up to nonsmooth optimization and generalized differential calculus. On the basis of the achieved results, he accomplished Doctor of Science degree (1991) and habilitated at Charles University (1997).

As an internationally recognized expert he has been serving as associated editor in SIAM Journal on Optimization (10 years), Set-Valued Analysis and Kybernetika. In 2007, he was awarded The Bernard Bolzano Honorary Medal for Merit in the Mathematical Sciences by the Czech Academy of Sciences and, in 2013, he became Adjunct Professor at the Federation University in Ballarat, Australia.

At the age of 70, Jiří Outrata still actively cooperates with several distinguished experts from the area of variational analysis and their joint results can be found in some top journals devoted to optimization.

Herewith, we congratulate Jiří Outrata on turning 70 and wish him a lot of energy both for ongoing scientific activities and for active sporting, namely his lifetime favorite sailing, skiing and mountain hiking.

paraoptXI organizers

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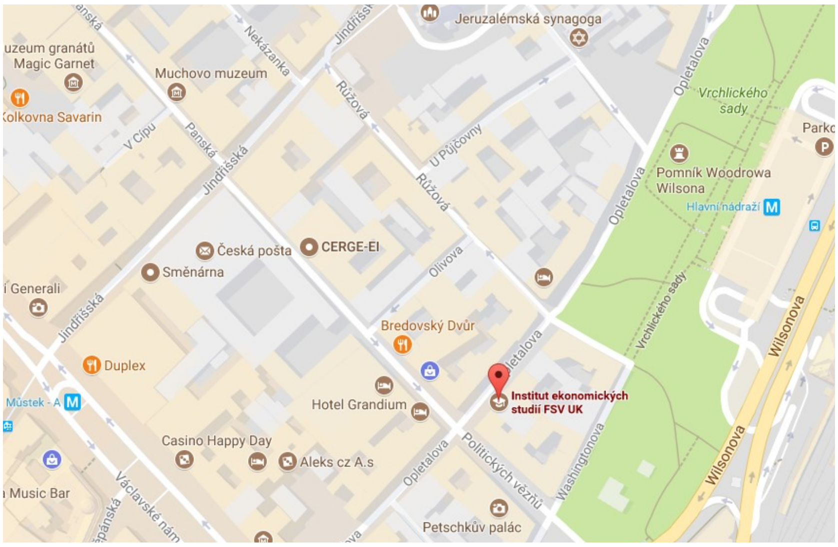


Conference Desk

The conference desk is situated in the Foyer of Lecture Hall 109.

Lecture Halls

Opening and Closing ceremonies and Plenary talks will take place in the Lecture Hall 109. Regular talks will take place in the Lecture Halls 206 and 314. To find your way to the lecture rooms, please follow the signs on site.

Presentation Instructions

Each lecture room is equipped with a computer and with a computer projector. You may also connect your own laptop.

The language of the conference is English only. Each regular presentation is 25 minutes including questions. Chairs are requested to keep the session on schedule. Papers should be presented in the order in which they are listed in the program for the convenience of attendees who may wish to switch rooms mid-session to hear particular presentations. In the case of a no-show, please use the extra time for a break or a discussion so that the remaining papers stay on schedule.

Computer Access

Eduroam Wi-Fi network is available at the conference venue. Additionally, login names and passwords for free Wi-Fi access shall be distributed upon request at the conference desk.

Lunch

Participants are asked to have their own lunches in nearby restaurants. A selection of recommended places is available at the conference desk.

Social Program

- Monday, September 18, 17:00 – 20:00 : informal **Welcome Reception** in Lecture Hall 109 with light snacks and drinks. The conference desk will be open (however, you may also pick up your conference materials anytime during the conference).
- Tuesday, September 19, 15:40 – 15:50 : Wellness Relaxation Break - Calming Pranayama, Anti-stress excercises, Energy Chime
- Wednesday, September 20, 15:40 – 15:50 : Wellness Relaxation Break - Natural Breathing, Strala Yoga, Koshi Water
- Wednesday, September 20, 18:00 – 20:00 : **Guided walking tour of Prague Castle & Lesser Town** - meeting point: at the lobby. You are going to see the famous Strahov courtyard and an amazing panoramic view of the whole city of Prague, then Hradčanské Square and all courtyards of Prague Castle with stunning St. Vitus' Cathedral, Old Royal Palace and Basilica of St. George. Continue down the stairs to Lesser Town, see a bit of Kampa island before crossing the famous Charles Bridge to the Old Town, where the tour ends.
- Thursday, September 21, 15:15 – 15:25 : Wellness Relaxation Break - Samavritti, Slowga, Sansula Harmony
- Thursday, September 21, 18:00 – 20:00 : **Guided walking tour of Old Town & New Town of Prague** - meeting point: at the lobby. You will see Wenceslas square with Jan Palach monument, National Theatre, the Charles Bridge, Kafka's house, the Estates' Theatre, the Old Town Square with the famous Astronomical Clock, Jewish Quarter with the Old-New Synagogue, Pařížská = Paris street built in Art Nouveau style, the tour ends at the boat.
- Thursday, September 21, 20:00: – 22:00 : **Conference Dinner** will be held on Taurus vessel. The ship is anchored near Čechův bridge - downstream, just in front of the hospital Na Františku (anchorage 11). See the map on the conference website http://paraoptxi.utia.cz/.

The social program is covered by the conference fee.

Post-conference Publications

There will be a special issue of *Optimization* dedicated to ParaoptXI. For this, all selected papers will have to pass a regular reviewing process. Details will be announced during the conference.

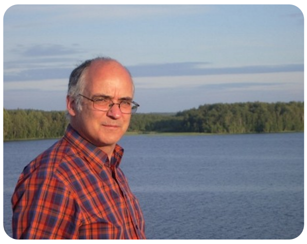
Date	Time	Lecture Hall 109	
		Lecture Hall 314	Lecture Hall 206
Monday September 18 2017	17:00 - 20:00	Welcome Reception + Registration	
Tuesday September 19 2017	08:30 - 09:00	Registration	
	09:00 - 09:15	Opening	
	09:15 - 10:15	René Henrion - A friendly tour through the world of calmness	
	10:15 - 10:45	Coffee break	
	10:45 - 12:00	Equilibrium and Disjunctive Problems	Projection Methods
	12:00 - 14:00	Lunch	
	14:00 - 15:40	Optimal Control	Stochastic Optimization
	15:40 - 16:10	Coffee break + Wellness Relax (10 min)	
	16:10 - 17:50	Generalized Nash Equilibrium Problems	Vector Programming
Wednesday September 20 2017	08:30 - 09:00	Registration	
	09:00 - 10:00	Alexander Kruger - Parametric Error Bounds and Metric Subregularity	
	10:00 - 10:30	Coffee break	
	10:30 - 12:10	Optimization in Infinite Dimensions	Bilevel Programs
	12:10 - 14:00	Lunch	
	14:00 - 15:40	Economics and Game Theory	Tangets, Normals and Subdifferentials
	15:40 - 16:10	Coffee break + Wellness Relax (10 min)	
	16:10 - 17:50	Disjunctive and Complementarity-type structures	Algorithms and Applications I
	18:00 - 20:00	Tour of Prague Castle	
Thursday September 21 2017	08:30 - 09:00	Registration	
	09:00 - 10:00	Michal Kočvara - New numerical tools for very large scale topology optimization	
	10:00 - 10:30	Coffee break	
	10:30 - 12:10	Subdifferentials and Directed Subdifferentials	Sensitivity and Stability
	12:10 - 14:00	Lunch	
	14:00 - 15:15	Mixed-Integer NLPs	Interval Data
	15:15 - 16:00	Coffee break + Wellness Relax (10 min)	
	16:00 - 17:00	Diethard Klatte - Parametric optimization and variational problems involving polyhedral multifunctions	
	18:00 - 20:00	Tour of Old Town	
	20:00 - 22:00	Boat reception	
Friday September 22 2017	08:30 - 09:00	Registration	
	09:00 - 10:15	Differential Inclusions	Calmness in Linear Programming
	10:15 - 10:45	Coffee break	
	10:45 - 12:00	Energy Markets	Algorithms and Applications II
	12:00 - 14:00	Lunch	
	14:00 - 15:15	Stability of Solution Maps	Local Topological Structures
	15:15 - 15:45	Coffee break	
	15:45 - 16:45	Boris Mordukhovich - Critical Multipliers in Parametric Variational Systems	
	16:45 - 17:00	Closing	

PLENARY TALK

Tuesday, 9:15 – 10:15, Lecture Hall 109

A friendly tour through the world of calmness

René Henrion, Weierstrass Institute Berlin, Germany



A major part of J. Outrata's mathematical work has been devoted to the investigation and application of the calmness property for multifunctions. This concept has turned out to be crucial in particular as a constraint qualification in order to derive M-stationarity conditions for MPECs etc. J. Outrata has made many significant contributions to the analysis of calmness, among them the highly important discovery that it may replace the much stronger Aubin property in the derivation of a crucial preimage formula for normal cones. The talk aims at an illustrative introduction of the calmness concept, at a reminiscence about the roots of common work with J. Outrata as well as a presentation of several more recent results on calmness issues. Among those figure a comparison of calmness between the canonical and enhanced perturbation mappings for generalized equations and estimations of the calmness modulus in linear and nonlinear programming.

EQUILIBRIUM AND DISJUNCTIVE PROBLEMS

Tuesday, 10:45 – 12:00, Lecture Hall 314

Computation of a Local Minimum of the MPCC

Tangi Migot, IRMAR-Insa Rennes, France
(joint work with *Dussault, J.-P. & Haddou, M & Kadrani, A.*)

Mathematical programs with complementarity constraints (MPCCs) are non-linear optimization problems over a possibly non-convex and non-connected domain. Recent progress on optimality conditions allows to build efficient relaxation methods that converge to MPCC-kind stationary point. However in 2015, Kanzow & Schwarz pointed out that the implementation of those methods may not guarantee anymore the strong convergence properties.

We introduce here an algorithm that tackle this issue by computing a specific approximate solution of the regularized sub-problems. We provide theoretical results on convergence and existence of the approximate solutions in a general framework that include all the relaxation methods from the literature. We also present numerical results in Julia.

Lagrange type duality and saddle point optimality criteria for mathematical programming problems with vanishing constraints

Yadvendra Singh, Banaras Hindu University, India
(joint work with *S. K. Mishra*)

In this paper, we consider a special class of optimization problem known as the mathematical problem with vanishing constraints (MPVC). We formulate the Lagrange type dual model for the MPVC and establish weak and strong duality results under generalized convexity assumptions. Further, we investigate the saddle point optimality criteria for the MPVC. We also illustrate our results by an example.

On the restrictiveness of sequential normal compactness in function spaces

Patrick Mehlitz, Technische Universität Bergakademie Freiberg, Germany

Sequential normal compactness (SNC) is one of the fundamental properties in modern variational analysis. Its presence is necessary for the derivation of the extremal principle in Banach spaces and, therefore, calculus rules for the computation of generalized normals to set intersections or preimages of sets under transformations. While the SNC-property is inherent in finite-dimensional Banach spaces, its validity has to be checked carefully in the infinite-dimensional setting.

In this talk, we first summarize some theoretical results on the SNC-property. Afterwards, we discuss whether or not sets of the type

$$\{y \in \mathscr{F}(\Omega;\mathbb{R}^q) \,|\, y(\omega) \in C(\omega) \text{ f.a.a. } \omega \in \Omega\},$$

which frequently appear in the context of control- or state-constrained optimal control, possess the SNC-property. Here, $\mathscr{F}(\Omega;\mathbb{R}^q)$ represents a Banach space of vector-valued functions defined on a domain $\Omega \subset \mathbb{R}^d$ equipped with Lebesgue's measure, e.g. $L^p(\Omega;\mathbb{R}^q)$ or $W^{1,p}(\Omega;\mathbb{R}^q)$ for $p \in (1,\infty)$, and $C\colon \Omega \rightrightarrows \mathbb{R}^q$ is a measurable set-valued mapping. Finally, we comment on the consequences of our results for the optimization community. Exemplarily, we focus on complementarity-constrained optimization problems in function spaces.

Projection Methods

Tuesday, 10:45 – 12:00, Lecture Hall 206

Modifying the Douglas–Rachford algorithm for solving best approximation problems

Rubén Campoy, University of Alicante, Spain
(joint work with *Francisco J. Aragón Artacho*)

In this talk we present a new iterative projection method for finding the closest point in the intersection of convex sets to any arbitrary point in a Hilbert space. This method, termed AAMR for averaged alternating modified reflections, can be viewed as an adequate modification of the Douglas–Rachford method that yields a solution to the best approximation problem. Under a constraint qualification at the point of interest, we show weak convergence of the method. In fact, the so-called strong CHIP fully characterizes the convergence of the AAMR method for every point in the space. The scheme is shown to be strongly convergent for affine constraints. We report some promising numerical experiments where we compare the performance of AAMR against other projection methods for finding the closest point in the intersection of pairs of finite dimensional subspaces.

A Direct Projection Method for the Block Kaczmarz Algorithm

Andrey Ivanov, Samara State Technical University, Russian
(joint work with *A.I. Zhdanov*)

This paper proposes a new implementation of the block Kaczmarz algorithm for solving systems of linear equations by the least squares method. Each iteration of the proposed algorithm can be considered as the solution of a sub-system defined by a specific arrowhead matrix. This sub-system is solved in an effective way using the direct projection method.

Main Results

For a matrix $A \in \mathbb{R}^{m\times n}$ and $f \in \mathbb{R}^m$ let $Au = f$ be a overdetermined consistent system of linear equation, where $u \in \mathbb{R}^n$ and $m \geq n$. Let's consider S as a positive-definite matrix (hence, invertible), whose Cholesky decomposition $S = LL^T$, where $S \in \mathbb{R}^{n\times n}$, $L \in \mathbb{R}^{n\times n}$. Consider matrix L as a right preconditioner, then we can write $Au = f$ as $AL\tilde{u} = f$ where $\tilde{u} = L^{-1}u$. To solve this system, we use the well-known Kaczmarz algorithm in a block modification. It's interesting that each iteration of this algorithm we can consider as

$$\begin{pmatrix} I_n & L^T A^T_{j(k)} \\ A_{j(k)} L & \left(1-\alpha_k^{-1}\right) A_{j(k)} S A^T_{j(k)} \end{pmatrix} \begin{pmatrix} \tilde{u}_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} \tilde{u}_k \\ f_{j(k)} \end{pmatrix} \Leftrightarrow Q_k x^k = b^k, \tag{1.1}$$

where

$$A = \begin{pmatrix} A_1 \\ \vdots \\ A_p \end{pmatrix}, \; f = \begin{pmatrix} f_1 \\ \vdots \\ f_p \end{pmatrix}, \; A_i = \begin{pmatrix} a^T_{(i-1)\cdot l+1} \\ \vdots \\ a^T_{(i-1)\cdot l+l} \end{pmatrix}, \; f_i = \begin{pmatrix} f_{(i-1)\cdot l+1} \\ \vdots \\ f_{(i-1)\cdot l+l} \end{pmatrix}$$

and $A_i \in \mathbb{R}^{l\times n}$, $f_i \in \mathbb{R}^l$, p is the number of blocks, l is the number of rows in the block A_i, $m = l \cdot p$, $i = 1, 2, \ldots, p$, and $j(k) : \mathbb{N}_0 \to \{1, 2, \ldots p\}$ is a surjection from a set of natural numbers with zero to a set of block indexes. We will assume that $\alpha_k \in (0, 2)$ for the convergence of iterations.

Theorem 1. *The iterations (1.1) are equivalent to the iterations of block Kaczmarz algorithm with a relaxation parameter α_k and can be written as:*

$$\begin{aligned} l < n: &\quad \tilde{u}_{k+1} = \tilde{u}_k - \alpha_k L^+_{j(k)} \left(L_{j(k)} \tilde{u}_k - f_{j(k)}\right), \\ l \geq n: &\quad u_{k+1} = u_k - \alpha_k A^+_{j(k)} \left(A_{j(k)} u_k - f_{j(k)}\right), \end{aligned}$$

where $L_{j(k)} = A_{j(k)}L$ and $\alpha_k \in (0,2)$, the $[\cdot]^+$ denotes the Moore-Penrose pseudoinverse, and $k = 0,1,\ldots,\infty$. In general, the proof is obvious but an especially interesting in the final step for $l \geq n$. We have to recall the famous results from [1] here, and we should note that $LL^+_{j(k)} = L\left(A_{j(k)}L\right)^+ = LL^+\left(A_{j(k)}LL^+\right)^+ = A^+_{j(k)}$.

Theorem 2. *The linear system $Q_k x^k = b^k$ is nonsingular and for any k $\det(Q_k) = \left(-\alpha_k^{-1}\right)^l \det\left(A_{j(k)}SA^T_{j(k)}\right)$.*
It's follow from Aitken block-diagonalization formula for Q_k and exploiting the fact that the determinant of the triangle matrix block is the product of the determinants of its diagonal blocks.

For solving linear system (1.1) at each iteration it is proposed to use the direct projection method [3, 2]. It is worth to note that the proposed matrix equation (1.1) has some interesting properties and can be solved using the well-known algorithms [4], some of them can be effective enough.

If the initial approximation is fulfilled for a especial matching conditions, then the first n iterations of the direct projection method are redundant. Moreover, we can assume that if $l \geq n$, then some of preconditioning techniques don't appear to be effective in any case.

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[1] Cline R.E. Note on the generalized inverse of the product of matrices. Siam Review. 1964 Jan 1;6(1):57.

[2] M. Benzi and C. D. Meyer, A direct projection method for sparse linear systems, SIAM Journal on Scientific Computing, 16(5)(1995), 1159–1176.

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On necessary conditions for linear convergence of fixed-point iterations

Hieu Thao Nguyen, Georg-August-Universität Göttingen, Germany
(joint work with *Russell Luke*)

Recent and ongoing developments on necessary conditions for linear convergence of fixed-point iterations will be discussed. As an application, various necessary and/or sufficient conditions for linear convergence of the alternating projection method are provided in both convex and nonconvex settings.

Optimal Control

Tuesday, 14:00 - 15:40, Lecture Hall 314

On solution of contact shape optimization problems with Coulomb friction and a solution-dependent friction coefficient

Petr Beremlijski, VSB - Technical University of Ostrava, Czech Republic
(joint work with *J. Haslinger, J. Outrata a R. Pathó*)

We are dealing with a discretized problem of shape optimization of 2D and 3D elastic body in unilateral contact, where the coefficient of friction is assumed to depend on the unknown solution. Mathematical modeling of the Coulomb friction problem leads to a system of finite-dimensional implicit variational inequalities parametrized by the so-called control variable, that describes the shape of the elastic body. It has been shown that if the coefficient of friction is Lipschitz and sufficiently small, then the discrete state problems are uniquely solvable for all admissible values of the control variable (the admissible set is assumed to be compact), and the state variables are Lipschitzian functions of the control variable.

The shape optimization problem belongs to a class of so-called mathematical programs with equilibrium constraints (MPECs). The uniqueness of the equilibria for fixed control variables enables us to apply the so-called implicit programming approach. Its main idea consists in minimization of a nonsmooth composite function generated by the objective and the (single-valued) control-state mapping. In our problem, the control-state mapping is much more complicated than in most MPECs solved so far in the literature, and the generalization of the relevant results is by no means straightforward. For the solution of this nonsmooth problem we use our Matlab implementation of bundle trust method proposed by Schramm and Zowe. In each step of the iteration process, we must be able to find the solution of the state problem (contact problem with Coulomb friction) and to compute one arbitrary Clarke subgradient.

To get subgradient information needed in the used numerical method we use the differential calculus of Mordukhovich. For solving a state problem we use the method of successive approximations. Each iterative step of the method requires us to solve the contact problem with given friction. As a result, we obtain a convex quadratic programming problem with a convex separable nonlinear inequality and linear equality constraints. For the solution of such problems we use a combination of inexact augmented Lagrangians in combination with active set based algorithms.

Numerical examples illustrate the efficiency and reliability of the suggested approach.

Optimal control problems with oscillations, concentrations, and discontinuities

Didier Henrion, LAAS-CNRS, France
(joint work with *Didier Henrion (LAAS-CNRS Univ. Toulouse, France and Czech Tech. Univ. Prague, Czechia), Martin Kružík (ÚTIA-AVČR Prague, Czechia), Tillmann Weisser (LAAS-CNRS Univ. Toulouse, France)*)

Optimal control problems with oscillation (chattering controls) and concentration (impulsive controls) can have integral performance criteria such that concentration of the control signal occurs at a discontinuity of the state signal. Techniques from functional analysis (extensions of DiPerna-Majda measures from the partial differential equations literature) are developed to give a precise meaning of the integral cost and to allow for the sound application of numerical methods. We show how this can be achieved for the Lasserre hierarchy of semidefinite programming relaxations. This includes in particular the use of compactification techniques allowing for unbounded time, state and control.

Global shape optimization methods based on surrogate models for the case of B-spline shape parameterization

Ivo Marinić-Kragić, FESB, University of Split, Croatia
(joint work with *Damir Vučina*)

Engineering shape optimization often involves computationally expensive numerical simulations such as computational fluid dynamics. Furthermore, these technical objects are almost always complex three-dimensional shapes that cannot adequately be described with a small number of shape variables. These problems are usually solved by gradient methods starting from an initial solution. This approach is computationally efficient especially when the gradients are calculated by the adjoint method. However, this approach can be used only to obtain a local optimum. To obtain the global optimal solution, a different approach is required. A possible path is to use B-spline surfaces to describe the shape and genetic algorithms for optimization of the B-spline control point coordinates. The problem with this approach is that many computationally expensive numerical simulations are required.

The objective of this paper is the evaluation of different surrogate models in cases of global shape optimization which involve B-spline shape parameterization and computational fluid dynamics. The first step is solving several selected problems using a genetic algorithm starting from a randomly generated population. The optimization variables are only the coordinates of the B-spline control points. After the optimal solution is obtained by a genetic algorithm, several surrogate models were tested in various stages of the optimization procedure. The results are used to propose which surrogate models are appropriate for different stages of the global optimization problem at hand. This leads to a surrogate based optimization method developed specifically for global shape optimization based on B-spline shape parameterization.

Parameter Identification in Elliptic Variational Inequalites

Joachim Gwinner, Universität der Bundeswehr München, Germany

This contribution is based on [3, 4, 5] and is concerned with the inverse problem of parameter identification in elliptic variational inequalities (VIs) of the first and second kind.

A prominent example of the latter class is the following *direct* problem: Find the function u in $H^1(\Omega) = \{v \in L^2(\Omega) : \nabla v \in (L^2(\Omega))^d\}$ on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d (d = 2,3)$ such that for any $v \in H^1(\Omega)$ there holds

$$\int_\Omega e(x)\ [\nabla u \cdot \nabla(v-u) + u(v-u)] + \int_{\partial\Omega} f(s)\ |u|(|v| - |u|) \geq \int_\Omega g(x)\ (v-u)\,. \tag{1.1}$$

This VI (1.1) provides a simplified scalar model of the Tresca frictional contact problem in linear elasticity. By the classic theory of variational inequalities there is a unique solution u of (1.1) if the datum g that enters the right-hand side is given in $L^2(\Omega)$ and moreover, the "ellipticity" parameter $e > 0$ in $L^\infty(\Omega)$ and the “friction” parameter $f > 0$ in $L^\infty(\partial\Omega)$ are known. Here we study the *inverse* problem that asks for the distributed parameters e and f, when the state u or, what is more realistic, some approximation $\tilde{u}$ from measurement is known. In other words, we are interested in the variable parameters e and f such that $u(e, f) = \tilde{u}$, what is however unrealistic due to the lack of regularity in the measured data. Consequently, the inverse problem of parameter identification will be posed as an optimization problem which aims to minimize the misfit function, namely the gap between the solution $u = u(e, f)$ and the measured data $\tilde{u}$. This approach has been precisely the case with simpler inverse problems. To the best of our knowledge, this is the first work on the inverse problem of parameter identification in variational inequalities that does not only treat the parameter e that is linked to a bilinear form here (or a linear form), but also the parameter f linked to a nonlinear nonsmooth function, like the modulus function above.

To cover this frictional contact model problem as well as other non-smooth problems from continuum mechanics we propose the following new abstract framework. Let (as above) V be a Hilbert space; moreover E, F Banach spaces with convex closed cones $E_+ \subset E$ and $F_+ \subset F$. Let as with [1], $t : E \times V \times V \to \mathbb{R}, (e,u,v) \mapsto t(e,u,v)$ a trilinear form and $l : V \to \mathbb{R}, v \mapsto l(v)$ a linear form. Assume that t is continuous such that $t(e,\cdot,\cdot)$ is $V-$elliptic for any fixed $e \in$ int E_+. Now in addition we have a "semisublinear form" $s : F \times V \to \mathbb{R}, (f,u) \mapsto s(f,u)$, that is, for any $u \in V$, $s(f,u)$ is linear in its first argument f on F and for any $f \in F_+$, $s(f,\cdot)$ is sublinear, continuous, and nonnegative on V. Moreover assume that $s(f, 0_V) = 0$ for any $f \in F$.

Then the forward problem is the following VI: Given $e \in$ int E_+ and $f \in F_+$, find $u \in V$ such that

$$t(e;u,v-u)\ +\ s(f;v)\ -\ s(f;u)\ \geq\ l(v-u)\,, \forall v \in V\,. \tag{1.2}$$

Now with given convex closed subsets $E^{ad} \subset$ int E_+ and $F^{ad} \subset F_+$ we seek to identify two parameters, namely the "ellipticity" parameter e in E^{ad} and the "friction" parameter f in F^{ad}.
In the first step we investigate the dependence of the solution of the forward problem on these parameters. We assume

$$t(e;u,v) \leq \bar{t}\ ||e||_E\ ||u||_V\ ||v||_V, \forall e \in E,\ u \in V,\ v \in V \tag{1.3}$$

$$t(e;u,u) \geq \underline{t}\ ||u||_V^2, \forall e \in E^{ad} \subset E,\ u \in V \tag{1.4}$$

$$|s(f;u_2) - s(f;u_1)| \leq \bar{s}\|f\|_F\|u_2 - u_1\|_V, \forall f \in F; u_1 \in V, u_2 \in V\,. \tag{1.5}$$

These assumptions can be verified in the model problem and further applications. Under these assumptions we obtain the following Lipschitz continuity result.
Theorem. Consider the uniquely defined solution map $(e,f) \in E^{ad} \times F^{ad} \mapsto u = S(e,f)$. Let $e_i \in E^{ad}, f_i \in F^{ad}$ $(i = 1,2)$. Then there holds for some constant $c > 0$

$$\|S(e_2,f_2) - S(e_1,f_1)\|_V \leq c\ \{\|e_1 - e_2\|_E\ +\ \|f_1 - f_2\|_F\}\,.$$

Then we present an optimization approach to the parameter identification problem as follows. Let an observation $\tilde{u} \in V$ be given. Then the parameter identification problem studied in this paper reads: Find parameters $e \in E^{ad}, f \in F^{ad}$ such that $u = S(e,f)$ minimizes the "misfit function" $j(e,f) := \frac{1}{2}\ \|S(e,f) - \tilde{u}\|^2$. Here we assume that the sought ellipticity and friction parameters are smooth enough to satisfy with compact imbeddings $E^{ad} \subset \hat{E} \subset\subset E; F^{ad} \subset \hat{F} \subset\subset F$. Thus with given weights $\beta > 0, \gamma > 0$ we pose the stabilized optimization problem

$$(OP) \qquad \text{minimize } j(e,f) + \frac{\beta}{2}\ \|e\|_{\hat{E}}^2 + \frac{\gamma}{2}\|f\|_{\hat{F}}^2$$

$$\text{subject to } e \in E^{ad}, f \in F^{ad}$$

Theorem. Suppose the above compact imbeddings. Suppose that the trilinear form t satisfies (1.3) and (1.4) and that the semisublinear form s satisfies (1.5). Then (OP) admits an optimal (not necessarily unique!) solution $(e^*, f^*, u) \in E^{ad} \times F^{ad} \times V$, where $u = S(e^*, f^*)$, i.e. $u \in V$ solves the VI (1.2).
Finally we turn to finite dimensional approximation in the optimization approach and establish a convergence result employing the Galerkin method and Mosco set convergence what can be realized by finite element, respectively boundary element methods; see e.g. [6, 2] for the forward problem.

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STOCHASTIC OPTIMIZATION

Tuesday, 14:00 - 15:40, Lecture Hall 206

Scenario Generation and Empirical Estimation in Stochastic Optimization Problems: Survey and Open Questions

Vlasta Kaňková, ÚTIA, Czech Academy of Sciences, Czech Republic

Economic andf financial processes are mostly influenced by a random factor and a decision parameter. The decision parameter can mostly be determined by an optimization problem depending on the probability measure. In applications mostly the decision parameter has to be obtained either on the data base (the underlying distribution is replaced by empirical one) or by the problem with a simpler (mostly discrete) distribution. In the both cases two problems correspond to the original one: real and approximate. The relationship between them has been studied mostly under "classical" assumptions: linear dependence on the measure, distributions with thin tails, "classical" constraints sets, independent random sample, one-objective problems and so on. Stability results have been mostly employed for it. An effort has arisen to relax these assumptions. The aim is to give a survey of results for these relaxed conditions, and to introduce problems waiting for solution.

Markov Decision Processes: Stability and Well-posedness.

Enrique Lemus-Rodríguez, Facultad de Ciencias Actuariales, Universidad Anáhuac México, Mexico

In this lecture, we analyze the concept of Well-posedness of Markov Decision Processes (MDP) optimality problems in the framework of Dontchev and Zolezzi regarding Hadamard and Tikhonov's definitions. Markov Decision Processes are well-known and important dynamic decision models of discrete-time stochastic processes studied at least since 1957 by R. Bellman (A Markovian Decision Process) with a wide range of applications in science and technology. Their stability has been studied in a sense close to well-posedness by E. Gordienko since at least the end of the eighties. We compare both approaches and using the extensive knowledge on well-posedness regarding optimal control problems we discuss explicitly their relationship.

On almost sure rates of convergence for sample average approximations

Ralf Werner, Augsburg University, Germany
(joint work with *Dirk Banholzer and Joerg Fliege*)

In this presentation we provide rates at which strongly consistent estimators in the sample average approximation approach (SAA) converge to their deterministic counterparts. To be able to quantify these rates at which convergence occurs in the almost sure sense, we consider the law of the iterated logarithm in a Banach space setting. We first establish convergence rates for the approximating objective functions under relatively mild assumptions. These rates can then be transferred to the estimators for optimal values and solutions of the approximated problem. Based on these results, we further show that under the same assumptions the SAA estimators converge in mean to their deterministic equivalents, at a rate which essentially coincides with the one in the almost sure sense. Eventually, we address the notion of convergence in probability and provide some weak convergence rates, albeit under very mild conditions.

Stability-based scenario generation in stochastic optimization

Werner Roemisch, Humboldt-University Berlin, Institute of Mathematics, Germany
(joint work with *R. Henrion*)

Scenarios are indispensable ingredients for the numerical solution of stochastic optimization problems. We review earlier stability-based approaches for scenario generation and suggest to make use of stability estimates based on distances containing minimal information. For linear two-stage stochastic programs the latter approach to scenario generation can be reformulated as best approximation problem for the expected recourse function and as generalized semi-infinite program, respectively. The latter model turns out to be convex if either right-hand sides or costs are random. Further properties and solution approaches are also discussed..

GENERALIZED NASH EQUILIBRIUM PROBLEMS

Tuesday, 16:10 - 17:50, Lecture Hall 314

A Generalized Nash Game for Computation Offloading with an Extension to an MPEC/EPEC Structure

Daniel Nowak, TU Darmstadt, Germany
(joint work with *Alexandra Schwartz*)

The number of tasks which are performed on wireless devices instead of stationary computers is steadily increasing. Due to limited resources on these mobile devices computation offloading became a relevant concept. We investigate a nonconvex generalized Nash game for a group of mobile users in which each user tries to minimize his own task completion time. For this the users can offload parts of their computation task to a connected cloudlet with limited computation power. Here, the time restriction resulting from offloading is formulated as a vanishing constraint. We show that a unique Nash equilibrium exists and that the price of anarchy is one, i.e. the equilibrium coincides with a centralized solution. Further we consider the possibility for some users to have a temporal advantage, which leads to an MPEC/EPEC structure.
This work is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt.

Solving linear generalized Nash equilibrium problems numerically

Axel Dreves, Universität der Bundeswehr München, Germany
(joint work with *Nathan Sudermann-Merx*)

We will discuss several numerical approaches for the solution of generalized Nash equilibrium problems where the cost and constraint functions are affine linear. Exploiting duality theory it is possible to design an algorithm that can compute the entire solution set and terminates after finite time. Practically this algorithm is effective for small dimensional problems only. For larger problems we have to be satisfied to compute some solutions. Since we have no second order derivatives we will provide new convergence criteria for a potential reduction algorithm. Then we will compare this algorithm with some projected gradient method and a penalty approach that exploit Nikaido-Isoda function based optimization reformulations. All algorithms are tested on randomly generated instances of some economy market models.

A Multiplier-Penalty-Method for Generalized Nash Equilibrium Problems in Banach Spaces

Christian Kanzow, University of Wuerzburg, Germany
(joint work with *Veronika Karl, Daniel Steck, Daniel Wachsmuth*)

We consider the (jointly convex) generalized Nash equilibrium problem (GNEP) in a Banach space setting. In order to find a suitable solution, we modify some recent ideas from infinite-dimensional optimization problems and suggest to use a multiplier-penalty-type scheme in order to solve the underlying GNEP. Under some mild and problem-tailored assumptions, we discuss topics like the existence of solutions, the well-definedness of the algorithm, as well as the feasibility and optimality of accumulation points. Some numerical results illustrate the efficiency of the overall scheme.

Dynamic boundary control game with a star of vibrating strings

Sonja Steffensen, RWTH Aachen, Germany
(joint work with *Martin Gugat*)

Consider a star-shaped network of vibrating strings. Each string is governed by the wave equation. At the central node, the states are coupled by algebraic node conditions in such a way that the energy is conserved. At each boundary node of the network there is a player that performs Dirichlet boundary control action and in this way influences the system state. We consider the corresponding antagonistic game, where each player minimizes her quadratic objective function that is the sum of a control cost and a tracking term for the final state. We show that under suitable assumptions a unique Nash equilibrium exists and give an explicit representation of the equilibrium strategies.

VECTOR PROGRAMMING

Tuesday, 16:10 - 17:50, Lecture Hall 206

Subdifferentials of nonlinear scalarization functions and applications

Bao Truong, Northern Michigan University, U.S.A.
(joint work with *Christiane Tammer, Marcus Hillmann*)

The aim of this talk is to provide some new results in computing Mordukhovich limiting subdifferentials of nonlinear scalarization functions in vector optimization with either fixed ordering cones or variable domination structures. As an application, efficient necessary optimization conditions for multiobjective problems are discussed.

Pascoletti-Serafini scalarization method and vector optimization problems with variable ordering structure

Morteza Oveisiha, Imam Khomeini International University, Iran

In optimization theory, we usually assume that the image space is partially ordered by a nontrivial convex cone. In 1974, Yu [4] introduced more general concepts of ordering structures in terms of domination structures. Recently, vector optimization problems with variable ordering structures are investigated intensively, see e.g. [1, 2, 3].

In this talk, some optimality concept for vector optimization problems with a variable ordering structure are considered. we present a generalization of Pascoletti-Serafini scalarization problem and give some necessary and sufficient conditions that a vector optimization problem has a (weak) nondominated element with respect to a variable ordering structure.

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Multiobjective variational problems in a geometric setting

Ariana Pitea, University Politehnica of Bucharest, Romania
(joint work with *Tadeusz Antczak*)

A class of multitime multiobjective variational problems with partial differential equations and/or inequations constraints in a geometric setting is studied. Necessary efficiency conditions and sufficient efficiency conditions are proved, by means of generalized convexities. Multiobjective variational dual problems are introduced and several duality results are established.

Vector optimization in real linear spaces: The role of core convex topology

Majid Soleimani-damaneh, University of Tehran, Iran
(joint work with *Ashkan Mohammadi [Wayne State University]*)

In this paper, the Core Convex Topology (CCT) on a real vector space X, as the strongest topology which makes X into a locally convex space, is reconstructed. It is shown that the algebraic interior and the vectorial closure notions, considered in the literature as replacements of topological interior and closure, respectively, in linear spaces without topology, are actually nothing else than the interior and closure w.r.t. the CCT. After reconstructing the CCT and investigating it, we show that the properties of this topology lead to directly extending various important results in vector optimization from topological vector spaces to real vector spaces.

Wednesday, 9:00 – 10:00, Lecture Hall 109

Parametric Error Bounds and Metric Subregularity

Alexander Kruger, Federation University Australia, Australia

I am going to demonstrate how (nonlinear) metric subregularity of set-valued mappings can be treated in the framework of the theory of linear error bounds of real-valued functions. For this purpose, the machinery of error bounds has been extended to functions defined on the product of two (metric or normed) spaces. A general classification scheme of necessary and sufficient conditions for the local error bound property/metric subregularity will be presented. Several kinds of primal space and subdifferential slopes for real-valued functions and set-valued mappings will be discussed.

OPTIMIZATION IN INFINITE DIMENSIONS

Wednesday, 10:30 - 12:10, Lecture Hall 314

Fenchel-Lagrange c-conjugate duality in infinite convex optimization

Jose Vidal, TU Chemnitz, Germany
(joint work with *Dr. María Dolores Fajardo Gómez*)

In this talk we obtain a Fenchel-Lagrange dual problem for an infinite dimensional optimization primal one, via perturbational approach and applying a conjugation scheme called c-conjugation.
Using this approach, Fenchel-Lagrange dual problem is shown to be a combination of Fenchel and Lagrange dual problems obtained in former works. Motivated by this relation, the purpose of this talk is twofold. First, we will analyse the main inequalities that these three optimization problems satisfy, as well as sufficient conditions for equality among their optimal values. Secondly, we will develop two closedness-type regularity conditions via epigraphs, and a characterization for strong Fenchel-Lagrange duality in terms of the infimal convolution of two functions. As it happens in the classical context with the lower semicontinuity and convexity of the involved functions in the primal problem, the evenly convexity and properness will be a compulsory requirement since the conjugation scheme that we are applying is appropriate for the class of evenly convex functions, which can be characterized as those functions whose epigraph can be expressed as the intersection of an arbitrary family of open half-spaces.
Finally, we extend such conditions to the study of stable strong Fenchel-Lagrange duality.

Convex smooth-like properties and Faces Radon-Nikodym property in Banach spaces

David Salas, Lab. PROMES CNRS, Université de Perpignan, France

We introduce the notion of convex smooth-like (resp. $w*$-smooth-like) properties, which are a generalization of the well-known Asplund (resp. $w*$-Asplund) property. These properties allow us to have different regularities for the subdifferential of convex functions. We show that many of the reductions made for Asplund property also work for these smooth-like properties. In this framework, we introduce a new geometrical property, called the Faces Radon-Nikodym property, and we prove that it is in duality with a convex $w*$-smooth-like property.

Existence results for mixed hemivariational-like inequalities involving set-valued maps

Nicusor Costea, Politehnica University of Bucharest, Romania
(joint work with *Ariana Pitea*)

Let X and Y be Banach spaces such that there exists a linear continuous operator $\gamma : X \to Y$ and let K be a nonempty, closed and convex subset of X. We study inequality problems of the type
Find $u \in K$ such that:

$$\exists u^* \in A(u): \ \langle u^*, \eta(v,u)\rangle + J^0(\gamma(u);\gamma(\eta(v,u))) \geq \psi(u,v), \forall v \in K,$$

where $A : X \to 2^{X^*}$ is a set-valued map, $J : Y \to \mathbb{R}$ is a locally Lipschitz functional, $\eta : X \times X \to X$ and $\psi : X \times X \to \mathbb{R}$ is a prescribed bifunction.
The presence of the set-valued map ensures that not one, but types of solution can be defined, while the presence of the bifunction does not allow the inequality to be equivalently written as an inclusion. We prove that the

inequality possesses at least one solution (strong solution) provided the set-valued map is upper semicontinuous (lower semicontinuous, respectively).

Characterizations of the Free Disposal Condition for Nonconvex Economies on Infinite Dimensional Commodity Spaces

Alejandro Jofré, Universidad de Chile, Chile
(joint work with *Abderrahim Jourani*)

Our aim in this paper is to prove geometric characterizations of the free disposal condition for nonconvex economies on infinite dimensional commodity spaces even if the cone and the production set involved in the condition have an empty interior such as in L1 with the positive cone L1+. We then use this characterization to prove the existence of Pareto and weak Pareto optimal points. We show that the free disposal hypothesis alone assures the extremality of the production set with respect to some set.

BILEVEL PROGRAMS

Wednesday, 10:30 - 12:10, Lecture Hall 206

Proximal approach in selection of subgame perfect Nash equilibria

Francesco Caruso, University of Naples Federico II, Italy
(joint work with *Maria Carmela Ceparano and Jacqueline Morgan*)

In one-leader one-follower two-stage games, multiplicity of Subgame Perfect Nash Equilibria (henceforth SPNE) arises when the optimal reaction of the follower to any choice of the leader is not always unique, i.e. when the best reply correspondence of the follower is not a single-valued map. This presentation concerns a new selection method for SPNE which makes use of a sequence of games designed using a proximal point algorithm, well-known optimization technique related to the so-called Moreau-Yosida regularization (Moreau 1965, Martinet 1972, Rockafellar 1976, Parikh and Boyd 2014 and references therein). Any game of the obtained sequence is a classical Stackelberg game (Von Stackelberg 1952), i.e. a one-leader one-follower two-stage game where the best reply correspondence of the follower is single-valued. This mechanism selection is in line with a previous one based on Tikhonov regularization, in Morgan and Patrone (2006), but using the class of proximal point algorithms has a twofold advantage: on the one hand, it can provide improvements in numerical implementations and, on the other hand, it has a clear interpretation: the follower payoff function is modified subtracting a term that can represent a physical and behavioural cost to move (Attouch and Soubeyran 2009). The constructive method and its effectiveness are illustrated and existence results for the selection are provided under mild assumptions on data, together with connections with other possible selection methods.

Simple Bilevel Programming and Extensions: Theory and Algorithms

Stephan Dempe, TU Bergakademie Freiberg, Germany
(joint work with *Nguyen Dinh, Joydeep Dutta, Tanushree Pandit*)

Let $S =$ argmin $\{h(x) : x \in C\}$ denote the solution set of a convex optimization problem, where $C \subset \mathbf{R}^n$ is a closed convex set and $f, h : \mathbf{R}^n \to \mathbf{R}$ are real-valued convex functions. The simple bilevel optimization problem is

$$\min\{f(x) : x \in S\}. \tag{1.1}$$

To investigate this convex optimization problem, the lower level problem needs to be transformed. If $\alpha = \min\{h(x) : x \in C\}$ denote the optimal value of this problem, it is equivalent to $\min\{f(x) : h(x) \leq \alpha,\ x \in C\}$. Slater's regularity condition is violated for this convex optimization problem. Using a variational inequality to express the set S, a simple MPEC

$$\min\{f(x) : x \in C,\ \psi_y(x) \leq 0\ \forall\ y \in C\} \tag{1.2}$$

arises, where $\psi_y(x) = \langle \nabla h(y), x - y \rangle$. (1.2) is a convex optimization problem. Results from semi-infinite optimization or the use of a gap function for the variational inequality can be applied to derive necessary and sufficient optimality conditions for (1.2) and, hence, for the original problem. One such optimality condition reads as:
A feasible point $\bar{x}$ is optimal for (1.2) if and only if there exist $k \in \mathbf{N}$, $\lambda_1, \lambda_2, \cdots, \lambda_k > 0$, $y_1, y_2, \cdots, y_k \in C$ such that

$$0 \in \partial f(\bar{x}) + \sum_{i=1}^{k} \lambda_i \nabla h(y_i) + N_C(\bar{x}) \text{ and} \langle \nabla h(y_i), \bar{x} - y_i \rangle = 0, \text{ for all } i = 1, 2, \cdots, k$$

provided some closedness qualification condition is satisfied and $\nabla h(x)$ is continuous and monotone.
In the second part of the talk, an idea for solving the problem will be given. Basis for this algorithm is a penalization $\xi_\varepsilon(x) = h(x) + \varepsilon f(x)$, where both f, h are assumed to be convex but not necessarily differentiable. The algorithm computes a sequence of η_k-optimal solutions of minimizing a Moreau-Yosida regularization of the function $\xi_\varepsilon(x)$ over C which converges to to an optimal solution provided this problem has a solution.

Optima and equilibria: new inner regularizations

Jacqueline Morgan, University of Naples Federico II, Italy
(joint work with *M.B. Lignola (University of Naples Federico II)*)

First, for a family of parametric minimum or equilibrium problems, we present new possible "inner regularizations" requiring, among other conditions, the lower semicontinuity of the approximation solution maps. Then, we propose corresponding concepts of "viscosity solutions" for bilevel problems (more precisely optimization or Minsup problems with constraints defined by a parametric minimum or equilibrium problem) for which existence of solutions, and/or nice asymptotic behavior of solutions under perturbations of the data, is not guaranteed.

A bilevel optimal control problem with PDEs

Felix Harder, TU Chemnitz, Germany
(joint work with *Gerd Wachsmuth*)

We consider a bilevel optimal control problem. The lower level problem is a classical optimal control problem with a parameterized objective function. The upper level problem is used to identify the (finitely many) parameters of the objective function of the lower level problem. We discuss assumptions for properties of the lower level problem such as directional differentiability of the parameter-to-solution map. In particular, we have to deal with the problem of two-norms discrepancy. We investigate optimality conditions for the bilevel problem.

ECONOMICS AND GAME THEORY

Wednesday, 14:00 - 15:40, Lecture Hall 314

Quadratic fractional programming under asymptotic analysis

Felipe Lara, Facultad de Ciencias, Universidad de Tarapacá, Chile
(joint work with *Alfredo Iusem*)

The quadratic fractional problem consists of minimizing a ratio of two functions, a quadratic function over an linear (affine) function. This problem has been mainly studied in the literature by its economics applications, like, among others, the minimization of cost/time or maximization of return/risk. In this talk, we use generalized asymptotic functions to deal with the quadratic fractional minimization problem. An extension of the Frank-Wolfe theorem from the quadratic to the quadratic fractional problem will be given. We established a characterization for the nonemptiness and compactness of the solution set in terms of the first and second order asymptotic functions. Finally, necessary and sufficient conditions for the particular cases of the linear fractional and the quadratic minimization problems are also provide.

Convolution, Non-transferable Utility, and Money

Sjur Didrik Flåm, University of Bergen, Norway

Convolution plays important roles in nonsmooth analysis and optimization. It does, however, not quite fit instances that feature "non-transferable utility." Such instances are common in economics and game theory. Additional problems emerge upon convoluting data or decisions that are distributed across diverse agents. This paper attempts to get around some of these hurdles by "monetizing" individual objectives and by using direct exchange as main vehicle.

Noncooperative Transportation Problem

Nathan Sudermann-Merx, BASF Business Services GmbH, Germany
(joint work with *Oliver Stein*)

The Noncooperative Transportation Problem (NTP) is a linear generalized Nash equlibrium problem where each player solves a classical transportation problem. We compute a set of Nash equilibria whose dimension deceeds the number of players by one. Further, we propose a subgradient method for the NTP and present an explicit formula for its subgradients. Finally, we suggest an optimization based treatment for the selection of a proper Nash equilibrium.

Sensitivity and stability in stochastic data envelopment analysis

Luka Neralić, Faculty of Economics and Business, University of Zagreb, Croatia
(joint work with *Rajiv D. Banker, Karlo Kotarac*)

Sensitivity and stability for Banker's model of Stochastic Data Envelopment Analysis (SDEA) is studied in this paper. In the case of the DEA model, necessary and sufficient conditions to preserve the efficiency of efficient Decision Making Units (DMUs) and the inefficiency of inefficient DMUs are obtained for different perturbations of data in the model. The cases of perturbations of all inputs, of perturbations of output and of the simultaneous perturbations of output and of all inputs are considered. An illustrative example is provided.

Tangets, Normals and Subdifferentials

Wednesday, 14:00 - 15:40, Lecture Hall 206

Calculus for directional limiting normal cones and subgradients

Matus Benko, Johannes Kepler University Linz, Austria
(joint work with *Helmut Gfrerer, Jiri Outrata*)

In the recent years, directional versions of the limiting (Mordukhovich) normal cone, the coderivative of a multifunction, the metric subregularity, etc. have been intensively studied by Gfrerer, yielding very interesting results. In this talk we present some basic calculus rules for these limiting objects valid under mild assumptions. E.g. we provide (upper) estimates for the directional limiting normal cone of a constraint set, the directional limiting subdifferential of a composition of functions, the directional coderivative of a composition of multifunctions, etc. We conclude the talk by showing some applications of the proposed calculus.

Restricted subdifferential and value function

Abderrahim Jourani, Institut de Mathématiques de Bourgogne, France

The aim is to introduce a subdifferential in the spirit of the limiting Fréchet subdifferential. Chain rules are established and several properties are given. We use this subdifferential to compute the subdifferential of subanalytic functions and the value function.

Sequential and exact formulae for the subdifferential of nonconvex integrand functionals

Pedro Pérez-Aros, Universidad de Chile - Universidad de O'Higgins, Chile
(joint work with *Rafael Correa and Abderrahim Hantoute*)

In this work we are interested in the study of generalized subdifferentials of the integral functional, formally defined in the form

$$I_f(x) := \int_T \max\{f(t,x),0\}d\mu(t) + \int_T \min\{f(t,x),0\}d\mu(t),\ \ x(\cdot) \in X,$$

with the associated normal integrand $f : T \times X \to \mathbb{R} \cup \{+\infty\}$, that is $f_t := f(t,\cdot)$ is lower semicontinuous and f measurable in both variables with respect to a complete σ-finite measure space $(T,\mathscr{A},\mu)$ and X is a Banach space.
The first part of the work concerns the study of bornological subdifferential of the integral functional I_f, which in particular covers the Hadammard, the Frechet and the Proximal subdifferentials. Basically we stablish that for every point $x^* \in \partial I_f(x)$ there are sequences of measurable selections $x_n^*(t) \in \partial f_t(x_n(t))$ for measurable function$x_n(\cdot)$ and $x_n^*(\cdot)$ close to the point x and x^*, respectively. This result is compared to the work of Ioffe [2] and Lopez-Thibault [1].
The final part of the work gives an upper-estimate for the Limiting/Mordukhovich subdifferential, the G-subdifferential of Ioffe and the Clarke-Rockafellar subdifferential of the integral function I_f including the non-Lipschitz case. For this purpose we use a generalized Lipschitz condition for the interchange of the subdifferential and the sign of integral.
Key words. normal integrand, generalized subdifferentials, variational principles.

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On primal regularity estimates for set-valued mappings

Marian Fabian, Mathematical Institute, Czech Academy of Sciences, Bohemia

We prove several regularity results for set-valued mappings. In some cases, we improve also the statements known already for single-valued mappings. Linear openness of the set-valued mapping in question is deduced from the properties of its suitable approximation. This approach goes back to the classical Lyusternik-Graves theorem saying that a continuously differentiable single-valued mapping between Banach spaces is linearly open around an interior point of its domain provided that its derivative at this point is surjective. In this paper, we consider approximations given by a graphical derivative, a contingent variation, a strict pseudo H-derivative, and a bunch of linear mappings.

DISJUNCTIVE AND COMPLEMENTARITY-TYPE STRUCTURES

Wednesday, 16:10 - 17:50, Lecture Hall 314

Regularization for a Complementarity Formulation of Cardinality Constrained Optimization Programs

Max Bucher, Technische Universität Darmstadt, Germany
(joint work with *Martin Branda, Michal Červinka, Alexandra Schwartz*)

Cardinality constraints are used to model the fact that the solution of an optimization problem is expected or desired to be sparse. They impose an upper bound on the cardinality of the support of feasible points. An objective function, possibly nonlinear, is to be minimized subject to the cardinality constraint as well as further nonlinear constraints. Applications of cardinality constraints include portfolio optimization, compressed sensing or the subset selection problem in regression.

Cardinality constrained optimization problems are difficult to solve, since the mapping of a vector to the cardinality of its support is a discontinuous function. Even testing feasibility is known to be NP-hard (Bienstock 1996). Solution techniques for cardinality constrained optimization problems include the reformulation as a mixed-integer nonlinear program or the approximation of the cardinality constraint with a l1 -norm.

We follow an approach by Burdakov, Kanzow and Schwartz which is a reformulation of the cardinality constraint with complementarity constraints using continuous auxiliary variables. This opens up the possibility to use methods from nonlinear optimization. The reformulation possesses a strong similarity to a mathematical program with complementarity constraints (MPCC) and, like an MPCC, does not fulfil standard constraint qualifications. We use the strong link between the aforementioned reformulation of cardinality constrained optimization problems and MPCCs to derive optimality conditions and numerical methods. Particularly we investigate second order optimality conditions for the reformulation. We then use these to derive new convergence results for a Scholtes-type regularization.

This work is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt Darmstadt.

Modeling Truss Structures using Vanishing and Cardinality Constraints

Alexandra Schwartz, TU Darmstadt, Germany
(joint work with *Anne Tabbert*)

When constructing a truss one can try to optimize its geometry with respect to certain objectives such as total weight. On the other hand, to ensure its stability under load, stress constraints have to be satisfied for those bars, which are realized in the optimal design. This can be modeled using vanishing constraints, see e.g. Achtziger, Kanzow 2008. However, the resulting designs often contain many thin bars, which may be due to numerical inaccuracies and is not practical to realize.

For this reason, we consider an extended formulation of the optimization problem with an additional cardinality constraint, which limits the number of bars that may be realized. We analyze the structure of the resulting optimization problem with vanishing and cardinality constraints with respect to necessary optimality conditions and suggest an adapted relaxation method. Numerical results for academic examples are presented.

Global optimization of generalized semi-infinite programs using disjunctive programming

Peter Kirst, Karlsruhe Institute of Technology, Germany
(joint work with *Oliver Stein*)

We propose a new branch-and-bound algorithm for global minimization of generalized semi-infinite programs. It treats the inherent disjunctive structure of these problems by tailored lower bounding procedures. Our numerical results on standard test problems with three different such lower bounding procedures show the merits of our approach.

On Mathematical Programs with Second-Order Cone Complementarity Constraints: Necessary Optimality Conditions and Constraint Qualifications

Jane Ye, University of Victoria, Canada
(joint work with *Jinchuan Zhou*)

In this talk we consider a mathematical program with second-order cone complementarity constraints (SOCMPCC). The SOCMPCC generalizes the mathematical program with complementarity constraints (MPCC) in replacing the set of nonnegative reals by second-order cones. There are difficulties in applying the classical Karush-Kuhn-Tucker condition to the SOCMPCC directly since the usual constraint qualification such as Robinson's constraint qualification never holds if it is considered as an optimization problem with a convex cone constraint. Using various reformulations and recent results on the exact formula for the proximal/regular and limiting normal cone, we derive necessary optimality conditions in the forms of the strong-, Mordukhovich- and Clarke- (S-, M- and C-) stationary conditions under certain constraint qualifications. We also show that unlike the MPCC, the classical KKT condition of the SOCMPCC is in general not equivalent to the S-stationary condition unless the dimension of each second-order cone is not more than two. Moreover we provide some verifiable constraint qualifications for the M-stationary condition to hold at a local minimum.

Algorithms and Applications I

Wednesday, 16:10 - 17:50, Lecture Hall 206

Parametric Optimization From the View of Computational Logic

Stefan Ratschan, Institute of Computer Science, Czech Academy of Sciences, Czech Republic

In the field of computational logic, computation with parameters has a long tradition. This can be related to parametric optimization by interpreting a parametric optimization problem

$$\min_{x\in\mathbb{R}^n} f(x,\theta), \\ g_1(x,\theta)\le 0,\dots,g_k(x,\theta)\le 0$$

as the problem of finding a witness function for the predicate logical formula

$$\exists x\,.\,g_1(x,\theta)\le 0\wedge\cdots\wedge g_k(x,\theta)\le 0\wedge \\ \forall y\,.\,[[g_1(y,\theta)\le 0\wedge\cdots\wedge g_k(y,\theta)\le 0]\Rightarrow f(y)\ge f(x)].$$

Starting from the theorem of A. Tarski (1951) that the theory of real-closed fields allows quantifier elimination, computational logic has produced a long thread of work on solving such formulas. This work has traditionally concentrated on the case where $f, g_1,\dots,g_k$ are polynomials, but recently there is also work on the non-polynomial case.
In the talk, I will give a non-comprehensive overview of such work, attempting to relate it to parametric optimization.

Adaptive Dynamic Subsampled Trust-Region Algorithm

Robert Mohr, Karlsruhe Institute of Technology, Germany
(joint work with *Oliver Stein*)

We propose a new algorithm for solving large-scale non-convex machine learning problems. The algorithm is based on the classical trust-region framework and incorporates gradient and hessian information of the objective function obtained through subsampling. The subsample size is adaptively increased during the algorithm. We present a theoretical convergence result and demonstrate on several test problems that our algorithm is competitive with state-of-the-art methods.

On the use of optimization techniques in the field of particle accelerators

Jean-Michel Lagniel, GANIL, France
(joint work with *Jean-Michel Lagniel GANIL, Caen, France*)

Particle accelerators are complex systems with a large number of tuning parameters (several hundreds) and many operational constraints (equipment operating ranges, limited beam losses to avoid the production of radiations and activations...). In this field optimization codes are used for beam physics studies (resolution of systems of nonlinear differential equations...) and also during the design studies to find the best accelerator structure to reach the desired performances minimizing both construction and operation costs. Optimization codes are also used during the operation phase to find “optimal commands”, the code acting directly on the accelerator equipments (magnets, rf cavities...) to optimize the accelerator performance (high beam brightness, low beam losses...). These different uses will be discussed taking examples from the SPIRAL-2 superconducting linear accelerator presently in its commissioning phase.

Liquid Handling Robot Scheduling in slide staining instrument for cancer diagnostics

Davaatseren Baatar, Monash University, Australia
(joint work with *Steven Edwards and Kate Smith-Miles*)

In this talk we consider a complex scheduling problem which arises in liquid handling robot scheduling in a slide staining instrument for cancer diagnostics. The problem can be considered as generalised scheduling problem due to the presence of almost all constraints considered in the scheduling literature and some additional new constraints such as “on-demand jobs” and “delayed” sequence dependent set-up time. The problem is a NP-hard, large scale, on-line scheduling problem with hard constraints. Finding a robust, efficient algorithm is becoming one of the main priorities of the manufactures of the instrument. We present a heuristics algorithm, which improves productivity of the instrument, reduces costs and increases reproducibility of the tests. Moreover, the scheduler can be used in designing of other instruments with the best resource utilisation outcomes. The algorithm is tested on real world data sets.

Thursday, 9:00 – 10:00, Lecture Hall 109

New numerical tools for very large scale topology optimization

Michal Kočvara, University of Birmingham, UK

The presentation starts with a brief introduction on topology optimization as a mathematical tool for optimal design of mechanical components. Although now routinely used in the industry, software for topology optimization suffers from limitations, in particular when used for complex three-dimensional structures. Several ways will be presented on how to substantially improve efficiency of topology optimization software using modern tools of numerical linear algebra and numerical optimization. These are based on domain decomposition and multigrid techniques and, for the more involved problems, on decomposition of large-scale matrix inequalities using recent results of graph theory.

SUBDIFFERENTIALS AND DIRECTED SUBDIFFERENTIALS

Thursday, 10:30 - 12:10, Lecture Hall 314

The Directed Subdifferential of DC and Quasidifferentiable Functions

Elza Farkhi, School of Math. Sciences, Tel Aviv University, Israel
(joint work with *Robert Baier, Vera Roshchina*)

Following an idea of A. Rubinov, the directed subdifferential of a difference of convex (DC) functions is defined as difference of the the two convex subdifferentials embedded in the space of directed sets. This approach is later applied to define directed subdifferential of quasidiffererentiable (QD) functions which have DC directional derivatives. Seven of the eight axioms of A. Ioffe for a subdifferential hold for the directed subdifferential, among them the exact sum rule (satisfied as equality). While preserving the most important properties of the quasidifferential, such as exact calculus rules, the directed subdifferential lacks the major drawbacks of quasidifferential: non-uniqueness and "inflation in size" of the two convex sets representing the quasidifferential after applying calculus rules. The visualization of the directed subdifferential is called the Rubinov subdifferential. The latter contains the Dini-Hadamard subdifferential as its convex part, the Dini-Hadamard superdifferential as its concave part, and its convex hull equals the Michel-Penot subdifferential. Hence, in general, the Rubinov subdifferential contains less critical points than the Michel-Penot subdifferential, while the sharp necessary and sufficient optimality conditions in terms of the Dini-Hadamard subdifferential are recovered by the convex part of the directed subdifferential. The directed subdifferential allows easily to detect and distinguish candidates for a maximum and for a minimum, as well as ascent and descent directions from its visualization.

Calculating the Nonconvex Rubinov Subdifferential for Generalized Quasidifferentiable Functions Without a D.C. Representation

Robert Baier, University of Bayreuth, Germany
(joint work with *Elza Farkhi, Vera Roshchina*)

The Rubinov subdifferential was introduced as the (usually nonconvex) visualization of the directed subdifferential, first for differences of convex (D.C.) and later for quasidifferentiable functions introduced by Demyanov/Rubinov. The directed subdifferential for both function classes is based on a certain arithmetic difference of the corresponding convex subdifferentials or quasisub-/quasisuperdifferential. Here, the convex sets are embedded in the Banach space of directed sets which allows an extension of the usual scalar multiplication and Minkowski sum of convex sets and offers a nonconvex visualization of differences of embedded convex sets.

Recently, the directed subdifferential is extended to the function class of directed subdifferentiable functions for which the function and certain restrictions to recursively defined orthogonal hyperplanes are uniformly directionally differentiable. This function class contains quasidifferentiable, amenable, lower-C^k and locally Lipschitz, definable functions on o-minimal structures. For a special subclass of D.C. functions in R^2 the Rubinov subdifferential coincides with the basic subdifferential of Mordukhovich offering thus a geometric calculation of the latter.

The directed subdifferential enjoys as good calculus rules as the quasidifferential, e.g., exact formulas for the sum or the maximum of functions. Due to properties of the visualization of directed sets, the Rubinov subdifferential contains further information on the subdifferentials of Dini, Michel-Penot, Clarke and Mordukhovich.

For simple examples with nonconvex visualizations, comparisons of this subdifferential with other known convex/nonconvex subdifferentials are shown in the context of necessary and sufficient optimality conditions of unconstrained optimization problems.

A Bundle Trust-Region Method for Marginal Functions Using Continuous Outer Subdifferentials

Martin Knossalla, University of Erlangen-Nürnberg, Germany

The theory of subdifferentials provides adequate methods and tools to put descent methods for nonsmooth optimization problems into practice. But in applications often exact information on the whole subdifferential is not available as, e.g. for marginal functions in parametric mathematical programming. In this situation the semismoothness of the objective function cannot be proved or is violated.
Based on continuous outer subdifferentials (COS) developed earlier, we present a constructive strategy for optimization problems with locally Lipschitz continuous objective function. At first this approach is described from a theoretical point of view and for arbitrary locally Lipschitz continuous functions. It is realized by projections onto the value of a COS. In practice, it may be that the computation of the whole COS is too difficult. For this reason we will construct and, further, approximate a COS for marginal functions. Based on this approximation a bundle trust-region method will is presented for which global convergence can be proved.

On Optimality Conditions for Constrained Nonsmooth Optimization Based on the Directed Subdifferential

Wolfgang Achtziger, Friedrich-Alexander University Erlangen-Nürnberg, Germany
(joint work with *R. Baier, E. Farkhi, V. Roshchina*)

We consider the directed subdifferential based on the concept of directed sets as suggested by Baier and Farkhi in 2008. The beauty of directed sets lies in the formal straightforwardness of the arithmetic calculus for embedded convex compact sets. As a consequence, direct and precise calculus rules apply for directed subdifferentials of sums, differences, and the pointwise maximum of nonsmooth functions. In the recent past necessary and sufficient optimality conditions based on the directed subdifferential have been suggested for unconstrained optimization of, e.g., quasidifferentiable functions. In this talk we present some results on standard approaches tackling constrained problems through optimality conditions of unconstrained optimization. We investigate the exact l1-penalty approach, Lagrange duality, and saddle point optimality conditions. The theoretical results are illustrated at hand of academic examples.

SENSITIVITY AND STABILITY

Thursday, 10:30 - 12:10, Lecture Hall 206

Computing Generalized Derivative Elements of Nonlinear Programs

Peter Stechlinski, MIT, United States
(joint work with *Amir Akbari, Johannes Jaschke, Kamil A. Khan, Paul I. Barton*)

New sensitivity analysis theory for parametric NLPs is presented; generalized derivative information is obtained for solutions of NLPs exhibiting active set changes. Two distinct approaches are considered: first, a nonsmooth implicit function theorem is applied to a nonsmooth NLP KKT system reformulation to yield sensitivity information under LICQ and SSOSC regularity conditions. This approach furnishes a nonsmooth sensitivity system that admits primal and dual sensitivities as its unique solution and recovers the classical results set forth by Fiacco and McCormick in the absence of active set changes. Next, multiparametric programming theory is applied to furnish limiting Jacobian elements of the primal variable solution. Here, the required regularity conditions are relaxed to MFCQ and CRCQ and GSSOSC; the set of multipliers is nonempty and bounded as linear dependence of constraints is permitted.

Both methods describe generalized derivative information in the form of elements of the limiting Jacobian (also called the B-subdifferential), which are computationally relevant objects in the sense that they imply attractive convergence properties of dedicated nonsmooth methods. Consequently, the newly created theory is amenable to tractable numerical implementations. This work is placed in the context of state-of-the-art sensitivity analysis theory for parametric NLPs and is motivated by nonlinear model predictive control problems.

Strong Stability of degenerated C–stationary points in MPCC

Daniel Hernandez Escobar, University of Bergen, Norway
(joint work with *Daniel Hernandez Escobar, Jan-Joachim Rückmann*)

We consider Mathematical Programs with Complementarity Constraints (MPCC). The study of strong stability of C-stationary points plays a relevant role in sensitivity analysis and parametric optimization. In 2010, H. Jongen et al. characterized the strong stability of C-stationary points for MPCC under the assumption that the Linear Independence Constraint Qualification (LICQ) holds. Here, we focus on the characterization of strong stability of C-stationary points when LICQ does not hold. We provide an upper bound on the number of constraints as a necessary condition for strong stability as well as a lower bound when a Mangasarian-Fromovitz-type constraint qualification does not hold. We introduce a weaker constraint qualification which turned out to be necessary for strong stability.

Sufficient conditions for Whitney differentiability of optimal-value functions for convex programs

Kamil Khan, McMaster University, Canada

In the spirit of the Whitney Extension Theorem, we consider a function on a compact subset of Euclidean space to be "Whitney differentiable" if it is a restriction of a continuously Fréchet-differentiable function with an open domain. Whitney differentiable functions have useful (yet possibly nonunique) derivatives even on the boundaries of their domains; these derivatives obey the usual chain rule and are valid subgradients if the function is convex. This presentation considers optimal-value functions for parametric convex programs whose constraints are translated by the parameters. Such functions may be nonsmooth; consider, for example, the mapping $p \in \mathbb{R}^2 \mapsto \min\{x \in \mathbb{R} : x \geq p_1, x \geq p_2\} = \max\{p_1, p_2\}$. Nevertheless, it is shown that when the objective and all constraints are Whitney-differentiable and a weakened variant of the linear-independence constraint qualification holds, then these optimal-value functions are guaranteed to be Whitney differentiable. This result extends classic sensitivity results for convex programs. Examples are presented, and implications are discussed for generating continuously differentiable convex underestimators of nonconvex functions for use in methods for deterministic global optimization.

Duality gap in linear optimization: stability and generic results

Miguel A. Goberna, University of Alicante, Spain
(joint work with *Andrea Ridolfi. A., Virginia N. Vera de Serio*)

We present recent stability results on the duality gap function g that measures the difference between the optimal values of the dual problem and of the primal problem in linear programming and in linear semi-infinite programming. In [1] we analyze the behavior of g when the data defining these problems may be perturbed, considering seven different scenarios. In particular we find some stability results by proving that, under mild conditions, either the duality gap of the perturbed problems is zero or $+\infty$ around the given data, or g has an infinite jump at it. We also give conditions guaranteeing that those data providing a finite duality gap are limits of sequences of data providing zero duality gap for sufficiently small perturbations, which is a generic result.

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MIXED-INTEGER NLPS

Thursday, 14:00 - 15:15, Lecture Hall 314

A Trust-Region-SQP Algorithm for the Efficient Solution of Non-Convex, Non-Relaxable Mixed-Integer Nonlinear Programming Problems

Klaus Schittkowski, University of Bayreuth, Germany
(joint work with *Oliver Exler, Thomas Lehmann*)

Mixed-integer optimization problems are often hard to solve, especially if function evaluations are very time-consuming or if integer variables are not relaxable. Based on the success of sequential quadratic programming (SQP) methods for continuous optimization, the idea to take integer variaables into account lead to a straigth-forward extension towards a mixed-integer nonlinear programming (MINLP) algorithm based on the SQP idea. The main design goal is to minimize the total number of function evaluations, i.e., executions of the underlying simulation software. The resulting code is called MISQP and and was developed at the Unversity of Bayreuth over a period of 10 years. A mixed-integer quadratic subproblem is solved in each step by a branch-and-bound method exploiting dual information. The algorithm is stabilized by a trust region correction which goes back to Yuan (1995). The Hessian of the Lagrangian function is approximated by BFGS updates subject to the continuous and the integer variables. We outline the mathematical structure of the algorithm and present some numerical results based on a set of 200 academic test examples.The number of function evaluations is about the same as the number of function evaluations needed to solve the relaxed test problems by a standard continuous SQP code (NLPQLP).

A feasible rounding approach for mixed-integer optimization problems: theoretical aspects

Oliver Stein, Karlsruhe Institute of Technology, Germany
(joint work with *Christoph Neumann, Nathan Sudermann-Merx*)

We introduce a new deterministic technique to generate good feasible points of mixed-integer optimization problems which satisfy a structural requirement that we call granularity. Finding a feasible point is known to be NP hard even for mixed-integer linear problems, so that many construction heuristics have been developed, among them the feasibility pump. In contrast to these heuristics, we show that solving certain purely continuous optimization problems and rounding their optimal points leads to feasible points of the original mixed-integer problem, as long as the latter is granular. The resulting algorithm is not heuristic but deterministic and, under an additional convexity assumption, even efficient.

The continuous optimization problems appearing in our approach have a structure that is similar to that of the continuous relaxation, and thus our approach has significant advantages over heuristics. For instance, the computational cost of our approach corresponds to merely a single step of the feasibility pump. For the objective function values of the generated feasible points we present computable a-priori and a-posteriori bounds on the deviation from the optimal value, as well as efficiently computable certificates for the granularity of a given problem.

A computational improvement step by integer line search and our promising experiences regarding the applicability of this approach are reported in the subsequent talk by Christoph Neumann.

A feasible rounding approach for mixed-integer optimization problems: algorithmic aspects and applications

Christoph Neumann, Karlsruhe Institute of Technology, Germany
(joint work with *Oliver Stein, Nathan Sudermann-Merx*)

A feasible rounding approach, introduced in the preceding talk by Oliver Stein, is a novel technique to compute good feasible points for mixed-integer optimization problems. The central idea of this approach is the construction of an enlarged inner parallel set for which any rounding of any point is feasible in the original problem. Under an additional convexity assumption, this approach may be carried out efficiently, if the optimization problem satisfies a structural requirement that is labeled granularity.

We elaborate that this concept is particularly promising for mixed-integer linear optimization problems due to the availability of an explicit and exact formula for the enlarged inner parallel set. We provide a computational study on optimization problems from the MIPLIB libraries and demonstrate that granularity may be expected in various real world applications. For these problems we show that our method is able to outperform standard software. Finally, we develop and test a post processing step using integer line search, which is able to significantly improve the computational results.

INTERVAL DATA

Thursday, 14:00 - 15:15, Lecture Hall 206

Tolerances, robustness and parametrization of matrix properties related to optimization problems

Milan Hladík, Faculty of Mathematics and Physics, Charles University, Czech Republic

When we speak about parametric programming, sensitivity analysis, or related topics, we usually mean the problem of studying specified perturbations of the data such that for a given optimization problem some optimality criterion remains satisfied. In this presentation, we turn to another question. Suppose that A is a matrix having a specific property P. What are the maximal allowable variations of the data such that the property still remains valid for the matrix? We study two basic forms of perturbations. The first is a perturbation in a given direction, which is closely related to parametric programming. The second type consists of all possible data variations in a neighbourhood specified by a certain matrix norm; this is related to the tolerance approach to sensitivity analysis, or to stability. The matrix properties that will be discussed are positive definiteness; P-matrix, H-matrix and P-matrix property; total positivity and inverse non-negativity.

Inverse interval linear programming problem

Amin Mostafaee, College of Science, Islamic Azad University Tehran, Iran
(joint work with *Milan Hladík*)

The current paper investigates inverse interval linear programming problems. Assume that the interval domains for the objective function and constraint coefficient are given. A certain value over the range of optimal values is available, we ask for which scenario this optimal value is attained. With respect to continuity of optimal value objective function, we provide a method by using the parametric linear programming techniques. We start with the case in which only the cost coefficients appear in the form of ranges. Then we proceed to the cases when the interval coefficients are situated in the objective function and/or in the right-hand side of the constraints. Finally, we study the generic case with possibly all coefficients affected by interval uncertainty.

Approximating the Optimal Value Range in Interval Linear Programming

Elif Garajová, Faculty of Mathematics and Physics, Charles University, Czech Republic
(joint work with *Milan Hladík*)

Interval linear programming provides a mathematical tool for modeling practical optimization problems affected by uncertainty. One of the essential tasks in optimization with interval-valued data is determining the range of optimal values of the objective function, i.e. finding the best and the worst possible optimal value. For general interval linear programs, this leads to an NP-hard problem. Therefore, methods for computing a quick and sufficiently tight approximation of these extremal optimal values are also desired. In this talk, we discuss the (in)approximability properties of this problem and propose a method for obtaining an approximation of the optimal value range. We also address some special classes of interval linear programs, for which the optimal value range can be computed exactly in polynomial time and study how the complexity of the problem varies with respect to the number and structure of interval coefficients in the given program.

Thursday, 16:00 – 17:00, Lecture Hall 109

Parametric optimization and variational problems involving polyhedral multifunctions

Diethard Klatte, University of Zurich, Switzerland

A multivalued mapping between finite dimensional spaces is called polyhedral (Robinson 1982) if its graph is the union of finitely many polyhedral convex sets. For example, the optimal solution set mapping of a "canonically perturbed" linear or quadratic program is polyhedral.

Similarly, the solution multifunction of an affine variational inequality under suitable parametrization has this property. Polyhedral multifunctions are of particular interest in the stability analysis of the classes of problems just mentioned and are also used in the study of linear inequalities, piecewise linear equations, complementarity problems, disjunctive optimization and many other subjects. The pioneering work in the 1970ies of F. Nozicka (a founder of the series of "Paraopt" conferences) on parametric linear optimization is one of the fundamentals for this theory. In our talk, we present characterizations and applications of several types of Lipschitz properties (e.g., upper Lipschitz behavior, metric regularity, strong regularity) for optimization and variational problems involving a polyhedral structure. We cover both classical and more recent developments on this topic. In particular, we show how to apply results from variational analysis to the classes of problems under consideration. This talk is co-authored by Bernd Kummer, Humboldt University Berlin.

DIFFERENTIAL INCLUSIONS

Friday, 09:00 - 10:15, Lecture Hall 314

Moreau-Yosida Regularization of State-Dependent Sweeping Processes with Nonregular Sets

Emilio Vilches, University of O'Higgins, Chile
(joint work with *A. Jourani*)

Let H be a separable Hilbert space. Given a set-valued map $C\colon [T_0,T]\times H \rightrightarrows H$ with nonempty closed values, the perturbed state-dependent sweeping process is the differential inclusion:

$$\begin{cases} -\dot{x}(t) \in N(C(t,x(t));x(t)) & \text{a.e. } t \in [T_0,T], \\ x(T_0) = x_0 \in C(T_0,x_0), \end{cases} \tag{1.1}$$

where for any subset $S \subset H$ the set $N(S;u)$ is the Clarke normal cone to S at $u \in S$. This differential inclusion is motivated by quasivariational inequalities arising, e.g., in the evolution of sandpiles, quasistatic evolution problems with friction, micromechanical damage models for iron materiales, etc.
To deal with (1.1), for $\lambda > 0$ let x_λ be any solution of the following penalized differential inclusion:

$$\begin{cases} -\dot{x}_\lambda(t) \in \dfrac{1}{2\lambda}\partial d^2_{C(t,x_\lambda(t))}(x_\lambda(t)) & \text{a.e. } t \in [T_0,T], \\ x_\lambda(T_0) = x_0 \in C(T_0). \end{cases} \tag{$\mathscr{P}_\lambda$}$$

In this talk, we show the convergence strongly pointwisely (up to a subsequence) of $(x_\lambda)_\lambda$ towards a solution of (1.1) when the moving sets are nonregular (subsmooth and positively α-far). Some relevant consequences are indicated.
Keywords: Sweeping process, Moreau-Yosida Regularization, Positively α-far sets, Subsmooth sets.

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Time optimal control: Application to LCL filters

Lukáš Adam, ÚTIA, Czech Academy of Sciences, Czech Republic
(joint work with *Václav Šmídl*)

We consider LCL filters which are important to remove unwanted frequencies or to enhance the wanted ones in a signal. Mathematically, we solve an optimal time control problem where we want to reach the prescribed state in minimal possible time. We rewrite the (controlled) differential equation into an (uncontrolled) differential inclusion and compute the reachable set based on convex duality. We discuss different viewpoints from mathematicians and engineers.

On Uniform Metric Regularity

Radek Cibulka, Faculty of Applied Sciences, University of West Bohemia, Czech Republic

We investigate uniform versions of metric regularity and strong metric regularity on compact subsets of Banach spaces, in particular, along continuous paths. These two properties turn out to play a key role in analyzing path-following schemes for tracking a solution trajectory of a parametric generalized equation or, more generally, of a differential generalized equation (DGE). The latter model covers a large territory in control and optimization, such as differential variational inequalities, control systems with constraints, as well as necessary optimality conditions in optimal control. We propose two inexact path-following methods for DGEs having the order of the grid error $O(h)$ and $O(h^2)$, respectively. We provide numerical experiments, comparing the schemes derived, for simple problems arising in physics. Further, we study metric regularity of mappings associated with a particular case of the DGE arising in control theory by focusing on the interplay between the pointwise versions of these properties and their infinite-dimensional counterparts.

CALMNESS IN LINEAR PROGRAMMING

Friday, 09:00 - 10:15, Lecture Hall 206

Point-based neighborhoods for sharp calmness constants in linear programming

Juan Parra López, Miguel Hernández University of Elche, Spain
(joint work with *M.J. Cánovas, J.-J- Rückmann, F.J. Toledo*)

This talk is focused on recent advances on the calmness property of ordinary (finite) linear programs under canonical perturbations (i.e., perturbations of the objective function coefficient vector and the right-hand side of the constraint system). We show that the expression for the calmness modulus of the argmin mapping given in [1] is indeed a calmness constant in a certain neighborhood, which is also provided, of the nominal minimizer. We emphasize the fact that the expressions for both the (sharp) calmness constant and the neighborhood can be easily computed as far as they only depend on the nominal data (in this sense we call them point-based). As an intermediate step, we show that an analogous fact occurs for the feasible set mapping associated with linear inequality systems under right-hand side perturbations: the point-based expression for the corresponding calmness modulus given in [2] turns out to be a calmness constant in a certain neighborhood, for which we also give a point-based expression. We also show that this result cannot be extended to general convex systems.

Key words. Calmness, feasible set mapping, optimal set mapping, linear programming, variational analysis.

Mathematics Subject Classification: 90C31, 49J53, 90C05, 49K40, 65F22.

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Calmness of partially perturbed linear systems. An application to the convergence analysis of the central path.

Marco Antonio López-Cerdá, Alicante University, Spain
(joint work with *M.J. Cánovas, J.A.J. Hall, J. Parra*)

In this talk we present point-based formulas for the calmness modulus of feasible set mappings associated with partially perturbed linear inequality systems. This is done exclusively in terms of the nominal problem's data, not involving data in a neighborhood, since the expressions for the calmness moduli are given in terms of such nominal data. In the second part, we show how the calmness modulus of a specific feasible set mapping constitutes an ingredient in the analysis of the linear convergence of the central path for a linear program. The result gives a new theoretical insight into not only the componentwise convergence of the complementarity products, but the componentwise convergence of primal and dual solutions in the context of non-degeneracy.

Calmness moduli from below and above of the optimal value function in linear programming

F. Javier Toledo Melero, Universidad Miguel Hernández de Elche, Spain
(joint work with *M.J. Gisbert Francés, M.J. Cánovas Cánovas, J. Parra López*)

The final goal of the paper presented in this talk consists in computing/estimating the calmness moduli from below and above of the optimal value function restricted to the set of solvable linear problems. Roughly speaking these moduli provide measures of the maximum rates of decrease and increase of the optimal value under perturbations of the data (provided that solvability is preserved). This research is developed in the framework of (finite) linear

optimization problems under canonical perturbations; i.e., under simultaneous perturbations of the right-hand-side (RHS) of the constraints and the coefficients of the objective function. As a first step, part of the work is developed in the context of RHS pertubations only, where a specific formulation for the optimal value function is provided. This formulation constitutes the starting point in providing exact formulae/estimations for the corresponding calmness moduli from below and above. We point out the fact that all expressions for the aimed calmness moduli are conceptually tractable (implementable) as far as they are given exclusively in terms of the nominal data.

Keywords. Calmness, Optimal Value, Linear programming

Mathematics Subject Classification: 90C31, 49J53, 49K40, 90C05

ENERGY MARKETS

Friday, 10:45 - 12:00, Lecture Hall 314

Recent advances in optimality conditions for nonlinear chance constrained problems with an application to energy sector

Martin Branda, ÚTIA, Czech Academy of Sciences, Czech Republic
(joint work with *Lukáš Adam*)

We deal with chance constrained problems (CCP) with differentiable nonlinear, possibly nonconvex, random functions and finite discrete distribution. First, we reformulate the problem using binary variables and discuss the drawbacks of this approach. By relaxing the binary variables we arrive at a nonlinear programming problem. We approach it as a mathematical program with complementarity constraints and discuss its relations to the original chance constrained problem with a special focus on the stationary points and (local) minima. Then, we regularize the relaxed problem by enlarging the set of feasible solutions using a regularization function. We derive necessary optimality conditions corresponding to the strong stationarity and discuss the convergence issues. We provide a numerical experiment comparing our approach with the integer programming one. We also discuss an application to gas network optimization.

Best Response of Producer in Pay-as-clear Electricity Market

Miroslav Pištěk, Institute of Information Theory and Automation (UTIA) of the Czech Academy of Sciences, Czech Republic
(joint work with *D. Aussel, R. Henrion, M. Červinka*)

We deal with several sources of uncertainty in electricity markets. The independent system operator (ISO) minimizes the production cost using chance constraints, thus hedging against discrepancies between estimated electricity demand and real consumption. We find an explicit solution of the ISO and use it to tackle the problem of a producer. In our model, producers face two sources of risk and uncertainty. Since producers participate in the day-ahead market, their productions, as well as revenues, are determined based on the estimate of electricity demand provided by the ISO. Thus, we consider producers to use chance constraints to hedge against the uncertainty of the ISO's prediction. Additionally, producers face uncertainty about the bid functions of their competitors. Thus each producer is maximizing his profit with respect to the worst case scenario, assuming the knowledge of the ambiguity sets in the form of intervals of bid-function coefficients of the competitors. To illustrate our results, we provide a numerical simulation of the producer's best response based on empirical data.

Short Term Forecasting for Electricity Demand in Egypt

Eman Mohamed, Faculty of Economics and Political Science, Cairo University, Egypt
(joint work with *Mohamed A. Ismail, Alyaa R. Zahran*)

Electricity is important for any nation. It influences not only the economy, but also the political and social aspects of a nation. Forecasting electricity demand is vital for future technical improvements. Short-term electricity demand forecasts are important for controlling of the electric power system. Recently, electricity demand series has found to contain more than one seasonal pattern. Intraday and intraweek seasonal patterns are appeared in the Egyptian electricity demand time series. This study investigates using Artificial Neural Networks in accommodating these seasonality patterns for forecasting hourly electricity demand in Egypt by using seasonal lags as inputs. Different artificial neural networks with different seasonal daily and weekly lags are used. The mean absolute percentage error is used to compare forecasting power of different artificial neural networks. Results indicate the accuracy of forecasts produced by the different artificial neural networks for different time horizons.

Keywords: Artificial neural networks, Double seasonality, Electricity demand forecasting, Mean absolute percentage.

ALGORITHMS AND APPLICATIONS II

Friday, 10:45 - 12:00, Lecture Hall 206

A gradient-based method for multiobjective optimization problems under uncertainties.

Quentin Mercier, ONERA, France
(joint work with *Fabrice Poirion, Jean-Antoine Désidéri*)

A method for solving multiobjective optimization problems where the objectives are given as expectations of random functions is proposed in this work. This method extends to the multiobjective case the classical stochastic gradient descent algorithm and therefore, on the contrary to classical approaches, it does not necessitate the costly estimation of the expectations obtained by using sample average approximation methods. The Stochastic Multiple Gradient Descent Algorithm (SMGDA) proposed here is based on the existence of a descent direction common to all the objectives. A recursive sequence is then built where at each step, the common descent vector is calculated from the updated objective function gradients. The step size is ruled by a sequence that has the same properties as the one in the classical stochastic gradient algorithm. Under a set of assumptions, both mean square and almost sure convergence can be proven. Illustrations of the method are given for different benchmark examples and compared to a more commonly used genetic algorithm (NSGA-II) coupled with a Monte Carlo estimator. The use of subgradients in the algorithm makes it able to converge towards the Pareto solutions even though the objectives are not differentiable. A final application under an engineering context will be provided, where a sandwich material is to be optimise under non-differentiable objectives and some geometrical constraints.

Primal-dual extragradient methods and stability of nonlinear nonsmooth PDE-constrained optimisation

Tuomo Valkonen, University of Liverpool, United Kingdom
(joint work with *Christian Clason (University Duisburg-Essen, Germany)*)

Using the metric regularity of infinite-dimensional set-valued mappings, we derive stability criteria for saddle points of nonsmooth optimization problems in Hilbert spaces. A main ingredient is an explicit pointwise characterization of the regular coderivative of the subdifferential of convex integral functionals. We apply these stability results to study the convergence of an extension of the Chambolle–Pock primal-dual algorithm to nonsmooth optimization problems involving nonlinear operators between function spaces. We show, both numerically and theoretically, the applicability of an accelerated variant of the algorithm to parameter identification problems for an elliptic partial differential equation with non-differentiable data fitting terms. Convergence is guaranteed if the data is finite-dimensional (with the unknown still in a function space), or a small amount of Moreau–Yosida regularisation is applied.

Convergence of nonsmooth descent methods via Kurdyka-Lojasiewicz inequality on Riemannian manifolds

Seyedehsomayeh Hosseini, University of Bonn, Germany

We examine convergence of subgradient-oriented descent methods in nonsmooth optimization on Riemannian manifolds. We prove convergence in the sense of subsequences for nonsmooth functions whose standard model is strict. Adding the Kurdyka-Lojasiewicz condition, we demonstrate convergence to a singular critical point.

Stability of Solution Maps

Friday, 14:00 - 15:15, Lecture Hall 314

Stability Analysis for Parameterized Equilibria with Conic Constraints

Helmut Gfrerer, Johannes Kepler University, Austria
(joint work with *Boris S. Mordukhovich*)

We consider parameterized equilibria governed by generalized equations whose multivalued parts are modeled via regular normals to nonconvex conic constraints. The main goal is to derive sufficient conditions for a certain Lipschitzian behavior of the solution maps like calmness or the Aubin property.

Graphical Derivatives and Stability Analysis for Parameterized Equilibria with Conic Constraints

Héctor Ramírez Cabrera, Universidad de Chile, Chile
(joint work with *Boris Mordukhovich and Jiří Outrata*)

The paper concerns parameterized equilibria governed by generalized equations whose multivalued parts are modeled via regular normals to nonconvex conic constraints. Our main goal is to derive a precise pointwise second-order formula for calculating the graphical derivative of the solution maps to such generalized equations that involves Lagrange multipliers of the corresponding KKT systems and critical cone directions. Then we apply the obtained formula to characterizing a Lipschitzian stability notion for the solution maps that is known as isolated calmness.

On two variants of calmness and their verification in a class of solution maps

Jiří Outrata, ÚTIA, Czech Academy of Sciences, Czech Republic
(joint work with *H.Gfrerer (J.Kepler Universitaet Linz)*)

The talk deals with two strengthened variants of the standard calmness property in connection with solution maps to a class of parameterized constraint- and variational systems. As to the first one, called isolated calmness, several new results from the second-order variational analysis will be presented. They facilitate the application of the Levy-Rockafellar criterion in case of examined solution maps. The second notion, which we term two-sided calmness, requires, in addition, a „reverse“ condition, which enables us to use this property, e.g., in post-optimal analysis. In the framework of an implicit (multi)function model a sufficient condition for this property will be provided and illustrated via a parameterized constraint system.

Local Topological Structures

Friday, 14:00 - 15:15, Lecture Hall 206

On the local representation of piecewise smooth equations as a Lipschitz manifold

Vladimir Shikhman, Technical University of Chemnitz, Germany
(joint work with *Dorsch, Dominik ; Jongen, Hubertus Th. ; Ruckmann, Jan-J.*)

We study systems of equation with n variables, given by k piecewise differentiable functions. The focus is on the representability of the solution set locally as an $(n-k)$ dimensional Lipschitz manifold. For that, nonsmooth versions of inverse function theorems are applied. It turns out that their applicability depends on the choice of a particular basis. To overcome this obstacle we introduce a strong full-rank assumption (SFRA) in terms of Clarke's generalized Jacobians. The SFRA claims the existence of a basis in which Clarke's inverse function theorem can be applied. Aiming at a characterization of SFRA, we consider also a full-rank assumption (FRA). The FRA insures the full rank of all matrices from the Clarke's generalized Jacobian. The article is devoted to the conjectured equivalence of SFRA and FRA. For min-type functions, we give reformulations of SFRA and FRA using orthogonal projections, basis enlargements, cross products, dual variables, as well as via exponentially many convex cones. The equivalence of SFRA and FRA is shown to be true for min-type functions in the new case $k = 3$.

Stationary Point Set: Topological Universality of Convex Quadratic Problems

Harald Günzel, RWTH Aachen University, Germany

We observe that the jet-space which is used to describe stationary points, Fritz-John-points and some basic constraint qualifications, is diffeomorphic to the product of state- and parameter-space of a natural (parametric) family of convex quadratic problems.
Since the diffeomorphism used to establish the latter property is just the corresponding jet-extension, this shows the topological universality of the quadratic family, i.e. topological properties of stationary point sets that are generically possible in general parametric optimization problems are already present in convex quadratic optimization.
We can use natural embeddings between stationary point sets and violation sets of the Mangasarian-Fromovitz constraint qualification (of different problem size) to make the universality theorem prove some other topological properties.

PLENARY TALK

Thursday, 15:45 – 16:45, Lecture Hall 109

Critical Multipliers in Parametric Variational Systems

Boris Mordukhovich, Wayne State University, USA

We introduce the notions of critical and noncritical multipliers for parametric variational systems, which include those arising in problems of nonlinear programming, composite and minimax constrained optimization, conic programming, etc. These notions extend the corresponding ones developed by Izmailov and Solodov for the classical KKT system. It has been well recognized that critical multipliers are largely responsible for slow convergence of major primal-dual algorithms of optimization. Based on advanced tools and results of second-order variational analysis and generalized differentiation, we obtain efficient characterizations of critical and noncritical multipliers in terms of the given data of the problems under consideration. Furthermore, it is shown that critical multipliers can be ruled out by full stability of local minimizers in various classes of composite optimization and conic programming problems. We also establish connections between noncriticality of multipliers and appropriate notions of calmness, which strongly relate to some recent developments by Jiří Outrata and his collaborators.

List of Participants

LIST OF CONTRIBUTIONS

Organized by:

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FACULTY
OF SOCIAL SCIENCES
Charles University

StarTour

Programme and Conference Committee Chairs:
Jan-J. Rückmann *University of Bergen, Norway*
Oliver Stein *Karlsruhe Institute of Technology, Germany*

Organizing Committee Chair:
Michal Červinka *ÚTIA, Czech Academy of Sciences, Prague, Czech Republic*

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