Parametric Optimization From the View of Computational Logic

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In the field of computational logic, computation with parameters has a long tradition. This can be related to parametric optimization by interpreting a parametric optimization problem

$$\min_{x \in \mathbb{R}^n} f(x, \theta),$$
$$g_1(x, \theta) \leq 0, \ldots, g_k(x, \theta) \leq 0$$

as the problem of finding a witness function for the predicate logical formula

$$\exists x \,.\, g_1(x, \theta) \leq 0 \wedge \cdots \wedge g_k(x, \theta) \leq 0 \wedge$$
$$\forall y \,.\, [[g_1(y, \theta) \leq 0 \wedge \cdots \wedge g_k(y, \theta) \leq 0] \Rightarrow f(y) \geq f(x)].$$

Starting from the theorem of A. Tarski (1951) that the theory of real-closed fields allows quantifier elimination, computational logic has produced a long thread of work on solving such formulas. This work has traditionally concentrated on the case where $f, g_1, \ldots, g_k$ are polynomials, but recently there is also work on the non-polynomial case.

In the talk, I will give a non-comprehensive overview of such work, attempting to relate it to parametric optimization.