Sequential and exact formulae for the subdifferential of nonconvex integrand functionals

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In this work we are interested in the study of generalized subdifferentials of the integral functional, formally defined in the form

$$I_f(x) := \int_T \max\{f(t,x),0\}d\mu(t) + \int_T \min\{f(t,x),0\}d\mu(t), \quad x(\cdot) \in X,$$

with the associated normal integrand $f : T \times X \to \mathbb{R} \cup \{+\infty\}$, that is $f_t := f(t,\cdot)$ is lower semicontinuous and f measurable in both variables with respect to a complete σ-finite measure space $(T, \mathcal{A}, \mu)$ and X is a Banach space.

The first part of the work concerns the study of bornological subdifferential of the integral functional I_f, which in particular covers the Hadammard, the Frechet and the Proximal subdifferentials. Basically we stablish that for every point $x^* \in \partial I_f(x)$ there are sequences of measurable selections $x_n^*(t) \in \partial f_t(x_n(t))$ for measurable function$x_n(\cdot)$ and $x_n^*(\cdot)$ close to the point x and x^*, respectively. This result is compared to the work of Ioffe [2] and Lopez-Thibault [1].

The final part of the work gives an upper-estimate for the Limiting/Mordukhovich subdifferential, the G-subdifferential of Ioffe and the Clarke-Rockafellar subdifferential of the integral function I_f including the non-Lipschitz case. For this purpose we use a generalized Lipschitz condition for the interchange of the subdifferential and the sign of integral.

Keywords: normal integrand, generalized subdifferentials, variational principles.

References

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[2] Ioffe, A. D., *Three theorems on subdifferentiation of convex integral functionals.* J. Convex Anal. 13 (2006), no. 3-4, 759–772.

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